09-05-07

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OIPE IAP20 SEP 04 2007 PATENT & TRADEMARK OFFICE

Attorney's Docket No. 042933/298965

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re:	Nativadade Lobo	Confirmation No.: 5615
Appl. No.:	09/625,201	
Filed:	July 21, 2000	
For:	PULSE SHAPING WHICH COMPENSATES FOR COMPONENT DISTORTION	

BOX ISSUE FEE
Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

SUBMITTAL OF PRIORITY DOCUMENT

To complete the requirements of 35 U.S.C. § 119, enclosed is a certified copy of Great Britain priority Application No. 9801308.9, filed January 21, 1998.

Respectfully submitted,

Cory C. Davis

Cory C. Davis
Registration No. 59,932

Customer No. 00826
Alston & Bird LLP
Bank of America Plaza
101 South Tryon Street, Suite 4000
Charlotte, NC 28280-4000
Tel Charlotte Office (704) 444-1000
Fax Charlotte Office (704) 444-1111

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Date of Deposit September 4, 2007

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Tamara Stevens

Tamara Stevens

tellectual
operty
fice

Fo ivity and Innovation

Concept House
Cardiff Road
Newport
South Wales
NP10 8QQ

I, the undersigned, being an officer duly authorised in accordance with Section 74(1) and (4) of the Deregulation & Contracting Out Act 1994, to sign and issue certificates on behalf of the Comptroller-General, hereby certify that annexed hereto is a true copy of the documents as originally filed in connection with patent application GB9801308.9 filed on 21 January 1998.

In accordance with the Patents (Companies Re-registration) Rules 1982, if a company named in this certificate and any accompanying documents has re-registered under the Companies Act 1980 with the same name as that with which it was registered immediately before re-egistration save for the substitution as, or inclusion as, the last part of the name of the words "public limited company" or their equivalents in Welsh, references to the name of the ompany in this certificate and any accompanying documents shall be treated as references to he name with which it is so re-registered.

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e-registration under the Companies Act does not constitute a new legal entity but merely company to certain additional company law rules.

Signed

Dated 21 August 2007

Patents Form 1/77

Patents Act 1977
(Rule 16)

The Patent Office

23JAN98 E332522-1 D02716
F01/7700 25.00 - 9801308.9

Request for grant of a patent

(See the notes on the back of this form. You can also get an explanatory leaflet from the Patent Office to help you fill in this form)

THE PATENT OFFICE
21 JAN 1998
LONDON

The Patent Office

Cardiff Road
Newport
Gwent NP9 1RH

1. Your reference

PAT 98001 GB

2. Patent application number *(The Patent Office will fill in this part)* 21 JAN 1998

9801308.9

3. Full name, address and postcode of the or of each applicant *(underline all surnames)*

NOKIA MOBILE PHONES LIMITED
KEILALAHDENTIE 4
02150 ESPOO
FINLAND

Patents ADP number *(if you know it)*

5911995004

If the applicant is a corporate body, give the country/state of its incorporation

FINLAND

4. Title of the invention

RECEIVER/MODULATOR

5. Name of your agent *(if you have one)*

MRS HELEN LOUISE HAWS

"Address for service" in the United Kingdom to which all correspondence should be sent *(including the postcode)*

NOKIA MOBILE PHONES
PATENT DEPARTMENT
ST GEORGES COURT
ST GEORGES ROAD
CAMBERLEY
SURREY GU15 3QZ UK

Patents ADP number *(if you know it)*

4105177001

6. If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and *(if you know it)* the or each application number

Country	Priority application number *(if you know it)*	Date of filing *(day / month / year)*

7. If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date of the earlier application

Number of earlier application	Date of filing *(day / month / year)*

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? *(Answer 'Yes' if:*
   - *a) any applicant named in part 3 is not an inventor, or*
   - *b) there is an inventor who is not named as an applicant, or*
   - *c) any named applicant is a corporate body.*

   See note (d))

YES

I certify this to be a true copy.
G D Court
Acting for Comptroller

9. Enter the number of sheets for any of the following items you are filing with this form. Do not count copies of the same document

Continuation sheets of this form	
Description	24
Claim*(s)*	4
Abstract	
Drawing*(s)*	IN WITH TEXT

10. If you are also filing any of the following, state how many against each item.

Priority documents	
Translations of priority documents	
Statement of inventorship and right to grant of a patent *(Patents Form 7/77)*	
Request for preliminary examination and search *(Patents Form 9/77)*	
Request for substantive examination *(Patents Form 10/77)*	
Any other documents *(please specify)*	

11. I/We request the grant of a patent on the basis of this application.

Signature H L Haws Date 21/1/98

H L HAWS AGENT FOR THE APPLICANT

12. Name and daytime telephone number of person to contact in the United Kingdom

HELEN HAWS 01276 419346

Warning

After an application for a patent has been filed, the Comptroller of the Patent Office will consider whether publication or communication of the invention should be prohibited or restricted under Section 22 of the Patents Act 1977. You will be informed if it is necessary to prohibit or restrict your invention in this way. Furthermore, if you live in the United Kingdom, Section 23 of the Patents Act 1977 stops you from applying for a patent abroad without first getting written permission from the Patent Office unless an application has been filed at least 6 weeks beforehand in the United Kingdom for a patent for the same invention and either no direction prohibiting publication or communication has been given, or any such direction has been revoked.

Notes

a) *If you need help to fill in this form or you have any questions, please contact the Patent Office on 0645 500505.*

b) *Write your answers in capital letters using black ink or you may type them.*

c) *If there is not enough space for all the relevant details on any part of this form, please continue on a separate sheet of paper and write "see continuation sheet" in the relevant part(s). Any continuation sheet should be attached to this form.*

d) *If you have answered 'Yes' Patents Form 7/77 will need to be filed.*

e) *Once you have filled in the form you must remember to sign and date it.*

f) *For details of the fee and ways to pay please contact the Patent Office.*

```
Needs["LaurentFunctions`"]

RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
   PhaseAngle[L_][t_] :→ (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).

Needs["LaurentNotationTest`"]

Needs::nocont : Context LaurentNotationTest` was not created when Needs was evaluated.
```

We develop the modulating function that is used in GSM first to use as an example to demonstrate Laurent's idea.

$$T := \frac{3}{812500}$$

$$BT := 0.3$$

$$\text{ModulationIndex} := \frac{1}{2}$$

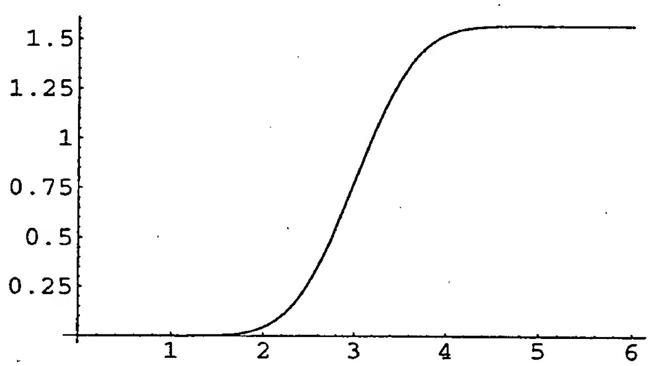
See LaurentTheory.nb to see the derivation of the C functions

When running the notebook we fix the value of L here and the rest of the graphs and functions use this value

For wide band CDMA applications
efficient operation of the PA occurs when it is saturated or
[illegible]
and thus power of hand held transmitters is a problem.
- By approximating to a constant envelope modulation we can mitigate the effects of the non-linear PA. We show that this can be done ~~without a penalty~~.

```
L := 6;
PhaseFunction[t_] = N[φL,t];
phasepoints = Table[{t, PhaseFunction[t T]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]
```

```
NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -1.91796×10^-7.

NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -5.52869×10^-7.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -1.96696×10^-7.

General::stop :
 Further output of NIntegrate::ncvb will be suppressed during this calculation.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

General::stop :
 Further output of NIntegrate::slwcon will be suppressed during this calculation.
```



- Graphics -

```
BitSeq = Table[1, {i, 1, 20}]
```

```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
RandomBitSeq = Table[Random[Integer, {0, 1}], {i, 1, 40}] // Map[# (-2) + 1&, #]&
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, 1, 1, 1, 1, -1, 1, -1, 1, 1, -1, 1, -1, -1, -1,
 -1, 1, 1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, -1, -1, -1, 1}
```

Testing The Modulator ...

Calculating the bandwidth of the signal

```
x11 = Table[LaurentC[L][0][t T], {t, 0, L + 1 - 1/200, 1/200}];
```

```
((Drop[x11, -1] - Drop[x11, 1]) 200 ) ^2 /200 // Apply[Plus, #]&
```

```
1.29684
```

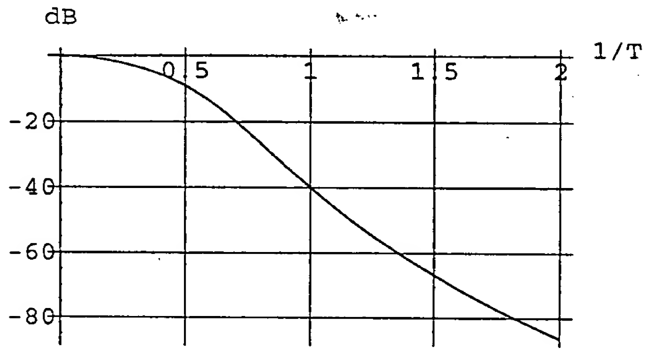
Here the pulse 6T is just stretched to 8T to see the performance

Example of stretching a C_0 pulse 0f 6 to 8

In this section we consider filtering the pulse

```
fil = BesselFilter[8][LowPass3dB[1 / T 2 Pi 0.3]][FrequencyResponse][s];
```

```
Plot[20 Log[10, Evaluate[(fil /. s → 2 Pi I f / T) // Abs]], {f, 0, 2},
 GridLines -> Automatic, AxesLabel -> {"1/T", "dB"}]
```

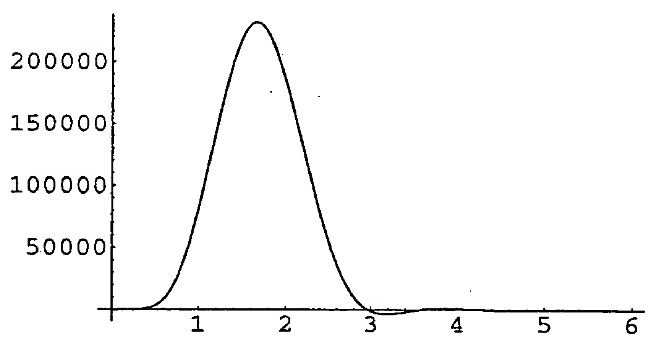


- Graphics -

```
Res[t_] = InverseLaplaceTransform[fil, s, t]
```

```
2027025 ((9.36812 + 0. I) E^(-897172. t) Cos[139301. t] -
   (13.7944 + 0. I) E^(-835672. t) Cos[420044. t] + (4.73111 + 0. I) E^(-701358. t) Cos[708768. t] -
   (0.304883 + 0. I) E^(-455818. t) Cos[1.02016×10^6 t] +
   (29.1264 + 0. I) E^(-897172. t) Sin[139301. t] -
   (9.89398 + 0. I) E^(-835672. t) Sin[420044. t] - (0.321205 + 0. I) E^(-701358. t) Sin[708768. t] +
   (0.375154 + 0. I) E^(-455818. t) Sin[1.02016×10^6 t])
```

```
Plot[Res[t T], {t, 0, 6}, PlotRange -> All]
```



- Graphics -

```
filTaps = Table[Res[t T], {t, 0, 6 , 1 / 20}] // Chop;
```

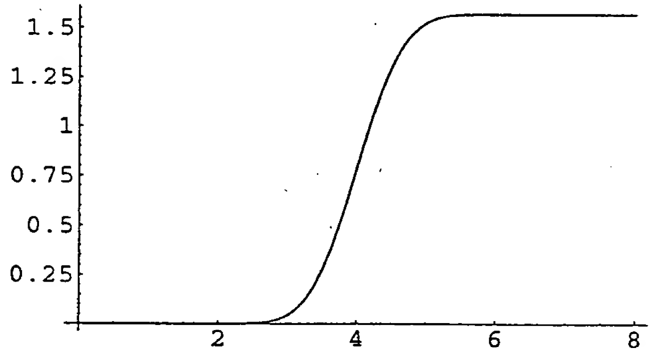
We estimate the group delay to be 1.8T

```
Length[filTaps]
```

121

```
L := 8;
PhaseFunction[t_] = N[φL,t];
phasepoints = Table[{t, PhaseFunction[t T]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]
```

```
NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -3.68196×10^-7.

NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -7.4914×10^-7.

NIntegrate::ncvb :
 NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections
   in LaurentFunctions`Private`t1 near LaurentFunctions`Private`t1 = -5.43232×10^-7.

General::stop :
 Further output of NIntegrate::ncvb will be suppressed during this calculation.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value
   of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
   If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.

General::stop :
 Further output of NIntegrate::slwcon will be suppressed during this calculation.
```



- Graphics -

```
pulse = Table[LaurentC[8][0][t T], {t, 0, 9, 1/20}];

pulse2 = Table[LaurentC[8][1][t T], {t, 0, 7, 1/20}];
```

FilteredPulse = T / 20 MapConvolve[pulse, filTaps] // Take[#, {34, Length[#] }]&

{1.73731×10^{-15}, 5.35887×10^{-15}, 1.60413×10^{-14}, 4.66278×10^{-14}, 1.31688×10^{-13},
3.61578×10^{-13}, 9.65729×10^{-13}, 2.51044×10^{-12}, 6.35507×10^{-12}, 1.56747×10^{-11},
3.76889×10^{-11}, 8.83867×10^{-11}, 2.02275×10^{-10}, 4.51958×10^{-10}, 9.86455×10^{-10},
2.10426×10^{-9}, 4.38922×10^{-9}, 8.95697×10^{-9}, 1.78914×10^{-8}, 3.4999×10^{-8}, 6.70842×10^{-8},
1.26054×10^{-7}, 2.32319×10^{-7}, 4.20166×10^{-7}, 7.46062×10^{-7}, 1.30124×10^{-6},
2.23035×10^{-6}, 3.75853×10^{-6}, 6.23×10^{-6}, 0.0000101618, 0.0000163174, 0.0000258049,
0.0000402068, 0.000061746, 0.000093496, 0.00013964, 0.000205786, 0.000299337,
0.000429924, 0.000609895, 0.000854853, 0.00118424, 0.00162196, 0.00219697,
0.0029439, 0.00390362, 0.00512368, 0.00665873, 0.00857068, 0.0109288, 0.0138093,
0.0172954, 0.021476, 0.0264451, 0.0323001, 0.0391407, 0.0470666, 0.0561754,
0.0665603, 0.0783079, 0.0914948, 0.106186, 0.12243, 0.140261, 0.159691, 0.180711,
0.20329, 0.22737, 0.25287, 0.279682, 0.307673, 0.336686, 0.366539, 0.397029,
0.427934, 0.459013, 0.490013, 0.520668, 0.550708, 0.579856, 0.607837, 0.634379,
0.65922, 0.682107, 0.702805, 0.721098, 0.73679, 0.749713, 0.759727, 0.76672,
0.770616, 0.771369, 0.768969, 0.763442, 0.754846, 0.743274, 0.728852, 0.711736,
0.692108, 0.670178, 0.646176, 0.62035, 0.592964, 0.56429, 0.534607, 0.504193,
0.473326, 0.442277, 0.411305, 0.380656, 0.350558, 0.321221, 0.29283, 0.26555,
0.239519, 0.214852, 0.191636, 0.169936, 0.149791, 0.13122, 0.114217, 0.0987613,
0.0848118, 0.0723143, 0.0612016, 0.0513967, 0.0428146, 0.035365, 0.0289542,
0.023487, 0.0188687, 0.0150065, 0.0118109, 0.00919671, 0.00708412, 0.0053992,
0.00407439, 0.00304877, 0.00226811, 0.0016848, 0.00125761, 0.000951417, 0.000736739,
0.000589309, 0.000489543, 0.000422011, 0.000374899, 0.000339478, 0.0003096,
0.000281223, 0.000251974, 0.000220764, 0.000187441, 0.000152497, 0.000116824,
0.0000815146, 0.0000477044, 0.0000164566, -0.0000113198, -0.0000349206,
-0.0000538729, -0.0000679391, -0.0000771062, -0.0000815622, -0.0000816651,
-0.0000779073, -0.000070877, -0.000061222, -0.0000496142, -0.0000367186,
-0.0000231668, -9.53584×10^{-6}, 3.66817×10^{-6}, 0.0000160205, 0.0000271838,
0.0000369093, 0.0000450345, 0.0000514769, 0.0000562268, 0.0000593363, 0.000060909,
0.0000610882, 0.0000600456, 0.0000579702, 0.0000550589, 0.0000515076, 0.0000475036,
0.0000432203, 0.000038813, 0.0000344153, 0.0000301387, 0.0000260716,
0.0000222801, 0.0000188096, 0.000015687, 0.0000129228, 0.000010514,
8.4469×10^{-6}, 6.69967×10^{-6}, 5.24481×10^{-6}, 4.05145×10^{-6}, 3.08723×10^{-6},
2.3199×10^{-6}, 1.71859×10^{-6}, 1.25466×10^{-6}, 9.0236×10^{-7}, 6.39109×10^{-7},
4.45602×10^{-7}, 3.05722×10^{-7}, 2.06318×10^{-7}, 1.36898×10^{-7}, 8.92721×10^{-8},
5.71866×10^{-8}, 3.59686×10^{-8}, 2.22017×10^{-8}, 1.34418×10^{-8}, 7.97812×10^{-9},
4.63953×10^{-9}, 2.64191×10^{-9}, 1.4722×10^{-9}, 8.02333×10^{-10}, 4.27388×10^{-10},
2.2234×10^{-10}, 1.12886×10^{-10}, 5.59276×10^{-11}, 2.70717×10^{-11}, 1.28211×10^{-11},
5.92683×10^{-12}, 2.65259×10^{-12}, 1.14156×10^{-12}, 4.73562×10^{-13}, 1.94329×10^{-13},
7.95416×10^{-14}, 3.14517×10^{-14}, 1.23089×10^{-14}, 4.73302×10^{-15}, 1.66595×10^{-15},
4.65283×10^{-16}, 1.93314×10^{-16}, 9.87738×10^{-17}, 2.31472×10^{-18}, -1.50823×10^{-17},
-1.01228×10^{-17}, -6.49877×10^{-18}, -2.08455×10^{-18}, 1.80177×10^{-19}, 1.70062×10^{-19},
1.9917×10^{-19}, -1.14008×10^{-19}, -1.66045×10^{-20}, 3.7877×10^{-21}, 1.47752×10^{-21},
1.94753×10^{-21}, 3.72298×10^{-22}, 2.02535×10^{-22}, 1.36007×10^{-22}, 6.72514×10^{-24},
7.07296×10^{-23}, 6.84862×10^{-23}, 3.68916×10^{-23}, 2.6013×10^{-24}, -1.51251×10^{-24},
-5.51867×10^{-25}, 7.52915×10^{-26}, 5.81584×10^{-25}, 1.75371×10^{-25}, 1.22392×10^{-26}, 0}

```
FilteredPulse2 = T / 20 MapConvolve[pulse2, filTaps] // Take[#, {50, Length[#] }]&
```

```
{1.27972×10^-7, 2.18306×10^-7, 3.66972×10^-7, 6.0806×10^-7, 9.93427×10^-7, 1.60076×10^-6,
 2.54471×10^-6, 3.99199×10^-6, 6.18149×10^-6, 9.45059×10^-6, 0.0000142688, 0.0000212804,
 0.0000313562, 0.0000456572, 0.0000657079, 0.0000934812, 0.000131493, 0.000182902,
 0.000251616, 0.000342391, 0.000460925, 0.000613932, 0.000809185, 0.00105553,
 0.00136282, 0.00174186, 0.00220418, 0.00276183, 0.00342705, 0.00421186, 0.00512763,
 0.00618453, 0.00739101, 0.00875319, 0.0102744, 0.0119544, 0.0137895, 0.0157714,
 0.0178876, 0.0201213, 0.0224509, 0.024851, 0.0272921, 0.0297419, 0.0321652,
 0.0345256, 0.0367857, 0.0389085, 0.0408581, 0.0426011, 0.044107, 0.0453496,
 0.0463073, 0.0469636, 0.0473081, 0.0473359, 0.0470483, 0.0464525, 0.0455611,
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 0.0306913, 0.0283283, 0.0259611, 0.0236189, 0.021328, 0.019112, 0.0169912,
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 0.0000507313, 0.0000318118, 0.0000253258, 0.0000271231, 0.0000338422, 0.0000428466,
 0.0000521516, 0.0000603451, 0.0000665053, 0.0000701193, 0.0000710042, 0.0000692325,
 0.0000650647, 0.0000588885, 0.0000511665, 0.0000423925, 0.0000330554, 0.0000236122,
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 -7.98457×10^-6, -5.18972×10^-6, -2.57416×10^-6, -2.1065×10^-7, 1.84943×10^-6,
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 5.41573×10^-6, 4.81423×10^-6, 4.20863×10^-6, 3.6208×10^-6, 3.06728×10^-6, 2.55958×10^-6,
 2.10471×10^-6, 1.70583×10^-6, 1.36297×10^-6, 1.07376×10^-6, 8.34162×10^-7, 6.3908×10^-7,
 4.82884×10^-7, 3.59854×10^-7, 2.64486×10^-7, 1.91715×10^-7, 1.37044×10^-7,
 9.65984×10^-8, 6.71324×10^-8, 4.59913×10^-8, 3.10539×10^-8, 2.06612×10^-8, 1.3542×10^-8,
 8.74155×10^-9, 5.55593×10^-9, 3.47564×10^-9, 2.13956×10^-9, 1.29613×10^-9,
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 4.40884×10^-11, 2.37341×10^-11, 1.26865×10^-11, 6.58958×10^-12, 3.34992×10^-12,
 1.6239×10^-12, 7.01037×10^-13, 2.33036×10^-13, 7.75051×10^-14, -3.22249×10^-15,
 -8.12875×10^-14, -7.89371×10^-14, -5.29088×10^-14, -3.14297×10^-14, -8.21475×10^-15,
 4.12557×10^-15, 1.9906×10^-15, 1.82068×10^-15, -4.36969×10^-15, 4.67143×10^-16,
 7.65162×10^-16, 5.67707×10^-16, 4.93342×10^-16, 7.52562×10^-17, 7.75606×10^-17,
 4.50401×10^-17, -2.18469×10^-17, 1.39012×10^-16, 1.92164×10^-16, 6.31946×10^-17,
 -6.72328×10^-18, 1.24704×10^-17, 1.42063×10^-17, 7.33335×10^-18, 1.13717×10^-19,
 3.90929×10^-19, 1.30285×10^-19, 0}
```
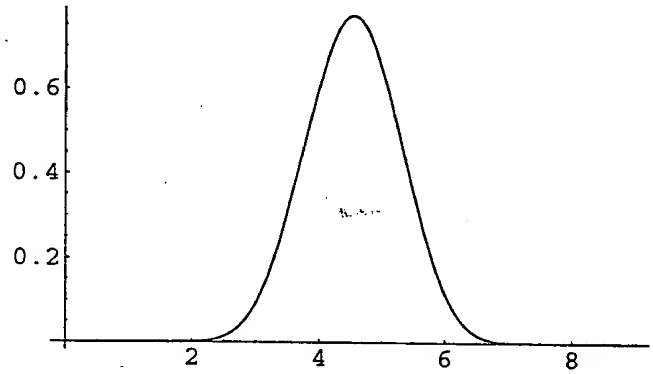

```
FiltPulse[8][0] = {(Table[t T, {t, 0, 20, 1/20}] // N) //
     Take[#, Length[FilteredPulse]]&, FilteredPulse} // Transpose //
  Interpolation
```

```
InterpolatingFunction[{{0, 0.0000492923}}, <>]
```

```
FiltPulse[8][1] = {(Table[t T, {t, 0, 20, 1/20}] // N) //
     Take[#, Length[FilteredPulse2]]&, FilteredPulse2} // Transpose //
  Interpolation
```

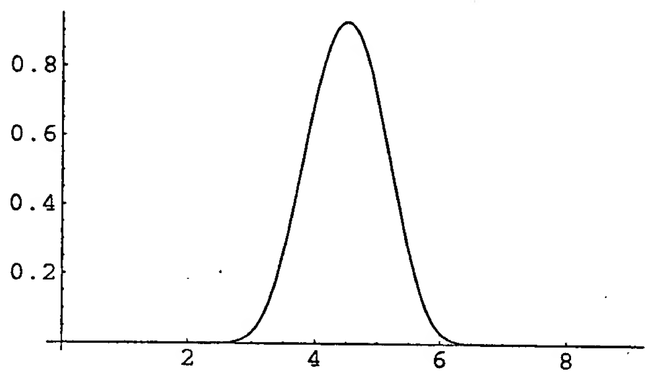
```
InterpolatingFunction[{{0, 0.0000389538}}, <>]
```

```
Plot[FiltPulse[8][0][t T], {t, 0, 9}, PlotRange -> All]
```



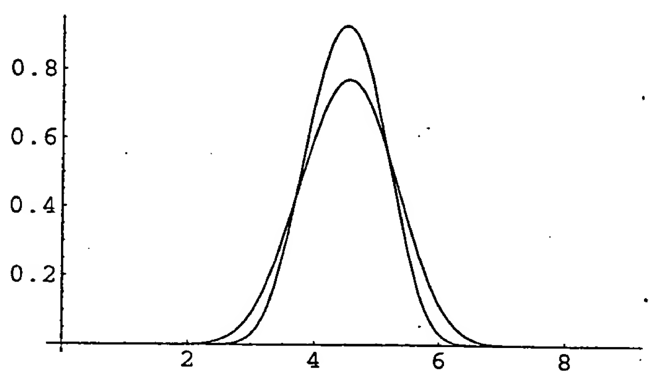
- Graphics -

```
Plot[LaurentC[8][0][t T], {t, 0, 9}, PlotRange -> All]
```



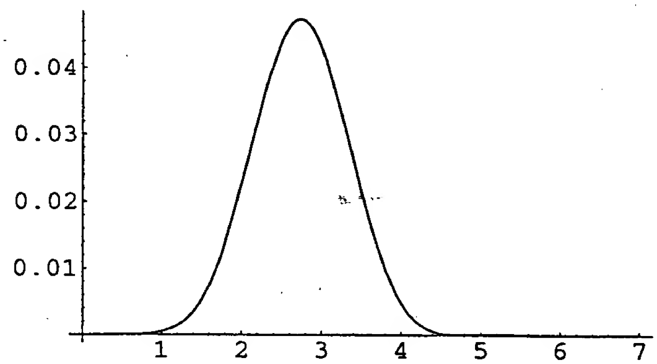
- Graphics -

```
Show[%, %%]
```



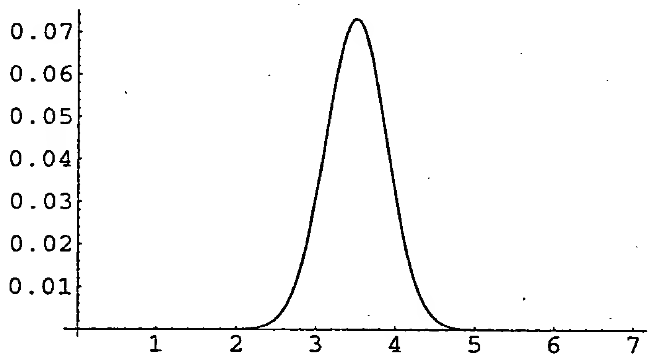
- Graphics -

```
Plot[FiltPulse[8][1][t T], {t, 0, 7}, PlotRange -> All]
```



- Graphics -

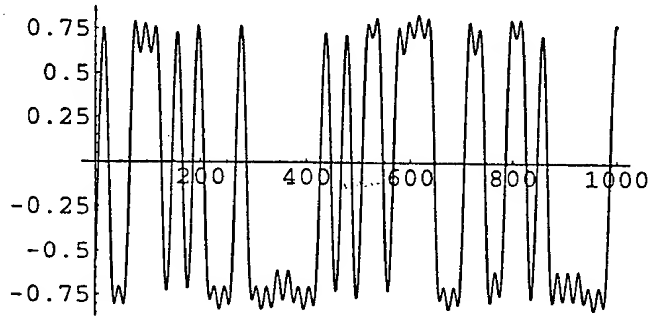
```
Plot[LaurentC[8][1][t T], {t, 0, 7}, PlotRange -> All]
```



- Graphics -

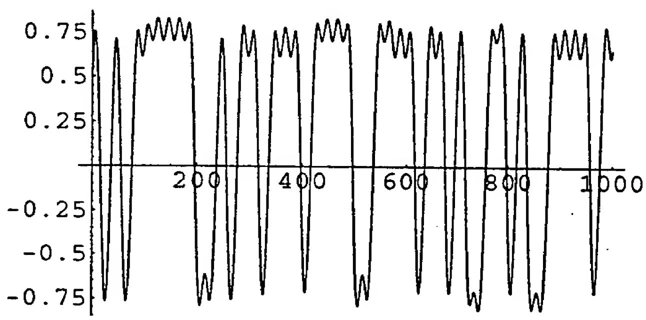
```
BandWidth[TestPulse_, Supp_] :=
 Module[{x11}, x11 = Table[TestPulse[t T], {t, 0, Supp - 1/200, 1/200}];
((Drop[x11, -1] - Drop[x11, 1]) 200 ) ^2 /200 // Apply[Plus, #]&]

BandWidth[FiltPulse[8][0], 9]

0.711659

BandWidth[LaurentC[8][0], 9]

1.29684

BandWidth[FiltPulse[8][1], 7]

0.00338446

tom = Modulator[L][Table[1, {i, 1, 100}], SamplingInterval → T/10,
  NumberOfCurves -> 2, ModulatingPulse → FiltPulse];

tom = Modulator[L][RandomBitSeq, SamplingInterval → T/10, NumberOfCurves -> 2,
  ModulatingPulse → FiltPulse];
```

```
ListPlot[Im[tom], PlotJoined -> True]
```



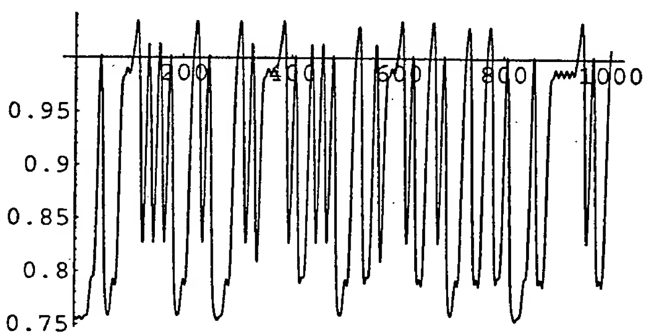
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



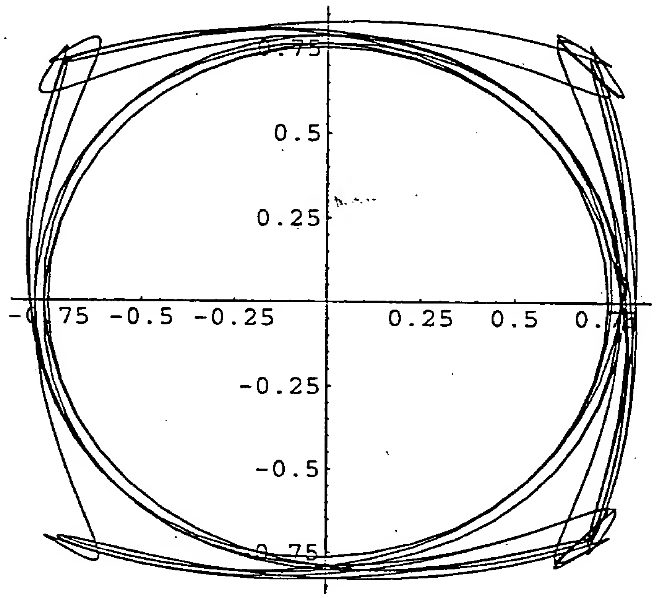
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

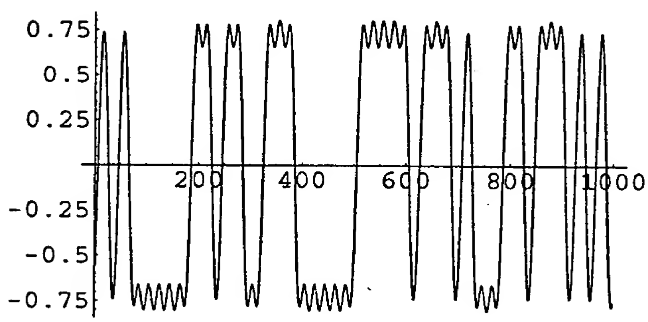
```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

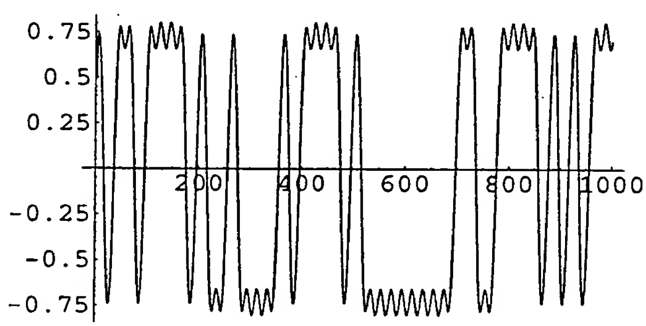
```
tom = Modulator[L][RandomBitSeq, SamplingInterval → T / 10, NumberOfCurves -> 1,
  ModulatingPulse → FiltPulse];
```

```
ListPlot[Im[tom], PlotJoined -> True]
```



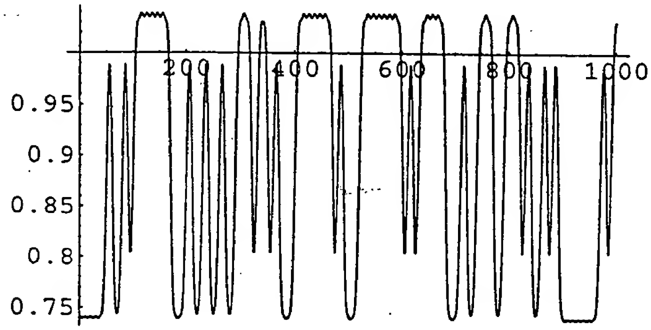
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



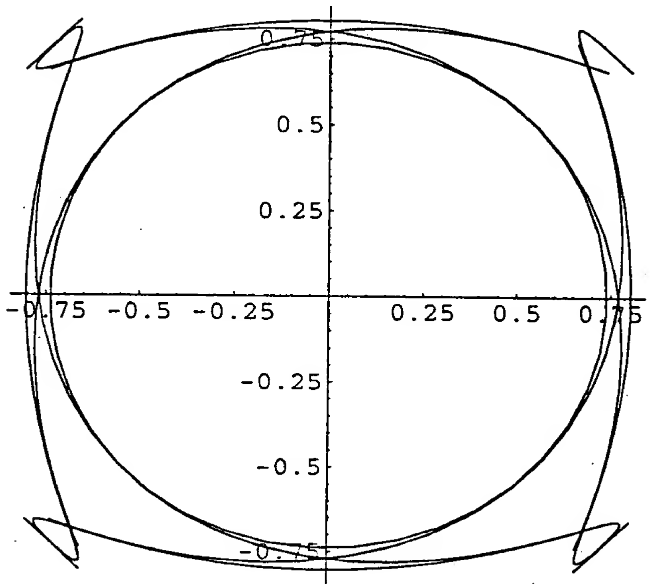
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

```
x11 = Table[TestPulse[6][0][t T], {t, 0, 7 - 1/200, 1/200}];

((Drop[x11, -1] - Drop[x11, 1]) 200 ) ^2 /200 // Apply[Plus, #]&
```

0.92632

```
TestPulse[6][0][t_] = LaurentC[4][0][t 5/7]
```

$$1.\ \mathrm{Sin}\left[\psi\left[4, \frac{3}{812500}+\frac{5\,t}{7}\right]\right] \mathrm{Sin}\left[\psi\left[4, \frac{3}{406250}+\frac{5\,t}{7}\right]\right] \mathrm{Sin}\left[\psi\left[4, \frac{9}{812500}+\frac{5\,t}{7}\right]\right] \mathrm{Sin}\left[\psi\left[4, \frac{5\,t}{7}\right]\right]$$

```
L := 5;
PhaseFunction[t_] = φL,t;
phasepoints = Table[{t, PhaseFunction[t T]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]
```

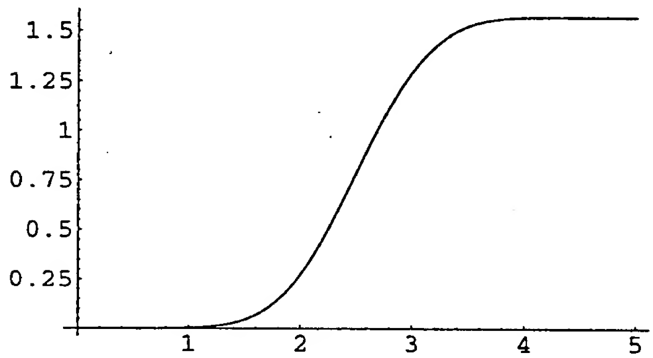
NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections in t1 near t1 = -1.48053×10^{-6}.

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections in t1 near t1 = -1.12255×10^{-6}.

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 7 recursive bisections in t1 near t1 = -1.08808×10^{-6}.

General::stop :
Further output of NIntegrate::ncvb will be suppressed during this calculation.

NIntegrate::slwcon : Numerical integration converging too slowly; suspect singularity, value of the integration is 0, oscillatory integrand, or insufficient WorkingPrecision.
If your integrand is oscillatory try using the option Method->Oscillatory in NIntegrate.



- Graphics -

```
L := 6
```

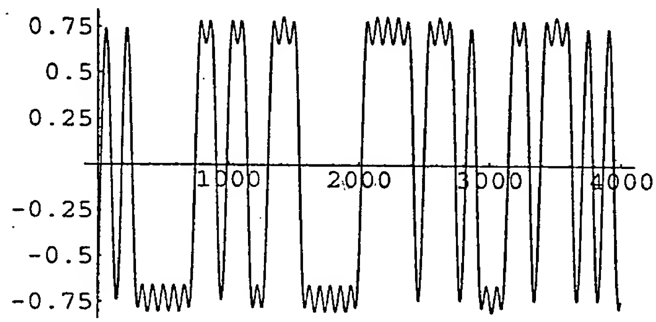
```
TestPulse2[6][0][t_] = LaurentC[4][0][t 6/7]
```

$$1.\,\mathrm{Sin}\left[\psi\left[4,\ \frac{3}{812500}+\frac{6\,t}{7}\right]\right]\ \mathrm{Sin}\left[\psi\left[4,\ \frac{3}{406250}+\frac{6\,t}{7}\right]\right]\ \mathrm{Sin}\left[\psi\left[4,\ \frac{9}{812500}+\frac{6\,t}{7}\right]\right]\ \mathrm{Sin}\left[\psi\left[4,\ \frac{6\,t}{7}\right]\right]$$

```
tom = Modulator[L][RandomBitSeq, SamplingInterval → T/40, NumberOfCurves -> 1,
  ModulatingPulse → FiltPulse];
```

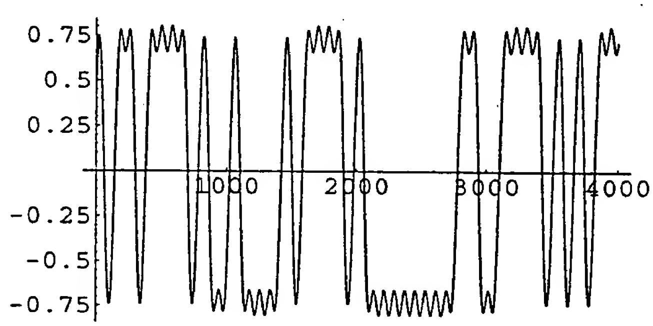
```
Save["ModulatorData.m", {T, L, RandomBitSeq, FiltPulse, tom}]
```

```
ListPlot[Im[tom], PlotJoined -> True]
```



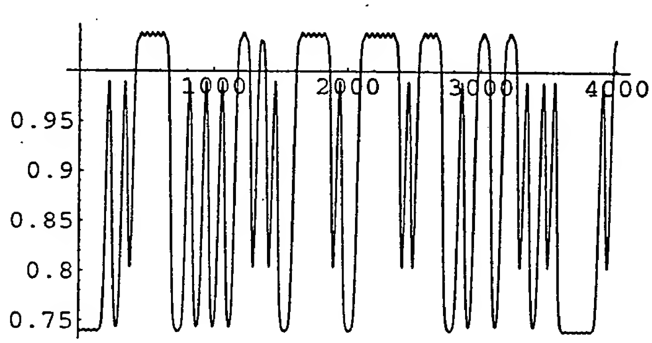
- Graphics -

```
ListPlot[Re[tom], PlotJoined -> True]
```



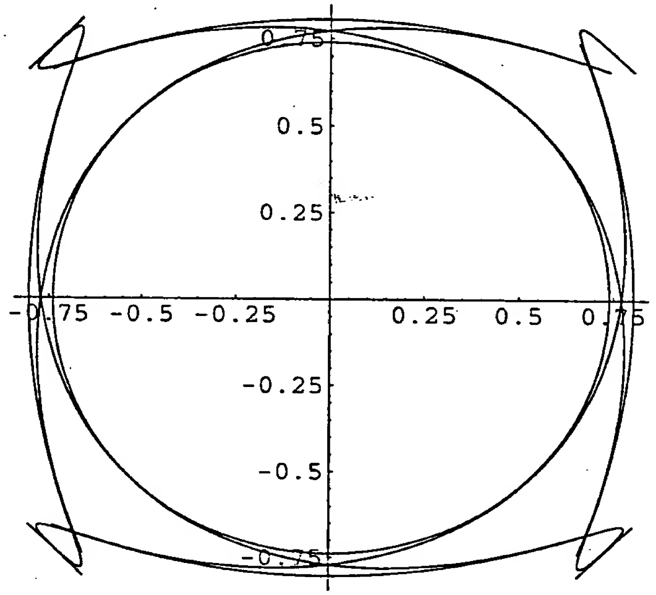
- Graphics -

```
ListPlot[Abs[tom], PlotJoined -> True]
```



- Graphics -

```
{Re[tom], Im[tom]} // Transpose // ListPlot[#, PlotJoined -> True, AspectRatio -> 1]&
```



- Graphics -

```
x11 = Table[TestPulse2[6][0][t T], {t, 0, 7 - 1/200, 1/200}];
```

```
Needs["LaurentFunctions`"]

RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
  PhaseAngle[L_][t_] :→ (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).

Needs["LaurentNotationTest`"]

Needs::nocont : Context LaurentNotationTest` was not created when Needs was evaluated.
```

Information on the functions used can be obtained using help.

```
Names["LaurentFunctions`*"]

{AKN, AlphaKI, ANKInitialStateSetUp, BT, FiltPulse, h, hFiltered, InitialState, J,
 LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
 NumberOfCurves, PhaseAngle, Receiver, ReceiverProper, S, SamplingInterval,
 StartingQuadrant, SyncSample, T, C, Φ, ψ}
```

$$T := \frac{3}{812500}$$

`BT := 0.3`

$$\text{ModulationIndex} := \frac{1}{2}$$

```
<< ModulatorData.m;

RandomBitSeq

{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

We show that it is possible to build a ~~receiver~~ mobile telephone to receive ~~pulses that have~~ a signal modulated using ~~pulse~~ supperposition method of Laurent - but with different pulses that have other desirable properties ~~like~~ like low bandwidth etc.

```
T
```

$$\frac{3}{812500}$$

$$S_{N\,T+\Delta T} = \sum_{K=0}^{M-1} \sum_{n'=0}^{L_K-1} J^{A_{K,N-n'}}\, C_{K,n'\,T+\Delta T}$$

With L = 8, and M = 2 we can utilise only the main values. It so happens that when K = 1, the dominant value occurs at 2.5T in the filtered pulse. The three dominant values when K =0 occurs at 3.5T, 4.5T and 5.5T. The value of the pulse at 6.5T is smaller than the value of the pulse with K = 1 at 2.5 T. Thus the expression becomes

$$S_{N\,T+\Delta T} = \sum_{K=0}^{1} \sum_{n'=3}^{5} J^{A_{K,N-n'}}\, C_{K,n'\,T+\Delta T}$$

$$\tilde{S}_{N\,T+\Delta T} = J^{\lambda_{0,N-3}}\,\mathtt{Pulse[0][3\,T+\delta T]} + J^{\lambda_{0,N-4}}\,\mathtt{Pulse[0][4\,T+\delta T]} + J^{\lambda_{0,N-5}}\,\mathtt{Pulse[0][5\,T+\delta T]} + J^{\lambda_{1,N-2}}\,\mathtt{Pulse[1][2\,T+\delta T]}$$

$$\tilde{S}_{N\,T+\Delta T} = J^{\lambda_{0,N-3}}\,\mathtt{Pulse[0][3\,T+\delta T]} + J^{\lambda_{0,N-4}}\,\mathtt{Pulse[0][4\,T+\delta T]} + J^{\lambda_{0,N-5}}\,\mathtt{Pulse[0][5\,T+\delta T]} + J^{(\lambda_{0,N-2}+\alpha_{N-3})}\,\mathtt{Pulse[1][2\,T+\delta T]}$$

Given that α_{N-3} has been already decided, we can precalculate the possibilities and store them.

$$\tilde{S}_{N\,T+\Delta T} = J^{\lambda_{0,N-5}}\,\left(J^{(\alpha_{N-4}+\alpha_{N-3})}\,\mathtt{Pulse[0][3\,T+\delta T]} + J^{\alpha_{N-4}}\,\mathtt{Pulse[0][4\,T+\delta T]} + \mathtt{Pulse[0][5\,T+\delta T]} + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-3})}\,\mathtt{Pulse[1][2\,T+\delta T]}\right)$$

But $J^2 = 1$ and so we get

$$\tilde{S}_{N\,T+\Delta T} = J^{\lambda_{0,N-5}}\,\left(J^{(\alpha_{N-4}+\alpha_{N-3})}\,\mathtt{Pulse[0][3\,T+\delta T]} + J^{\alpha_{N-4}}\,\mathtt{Pulse[0][4\,T+\delta T]} + \mathtt{Pulse[0][5\,T+\delta T]} + J^{(\alpha_{N-4}+\alpha_{N-2})}\,\mathtt{Pulse[1][2\,T+\delta T]}\right)$$

```
LookUpTable[Pulse_, δT_] :=
    Module[{x1, x2, x3, x4},
  x1 = {{-1, -1, -1}, {-1, -1, 1},
     {-1, 1, -1}, {-1, 1, 1}, {1, -1, -1}, {1, -1, 1}, {1, 1, -1}, {1, 1, 1}};
  x2[bitseq_] :=
   ( J^(bitseq[[3]] + bitseq[[2]]) Pulse[0][3 T + δT] + J^bitseq[[3]] Pulse[0][4 T + δT] +
     Pulse[0][5 T + δT] + J^(bitseq[[3]] + bitseq[[1]]) Pulse[1][2 T + δT]);
 {x1, Map[x2[#]&, x1]} // Transpose]
```

```
tab = LookUpTable[FiltPulse[8], 0.5 T]
```

```
{{{-1, -1, -1}, -0.00122183 - 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
 {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, 1, 1}, 0.0869922 + 0.770616 I},
 {{1, -1, -1}, 0.0869922 - 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
 {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, 1, 1}, -0.00122183 + 0.770616 I}}
```

```
Sort[tab, #1[[1, 3]] > #2[[1, 3]]&]
```

```
{{{1, 1, 1}, -0.00122183 + 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
 {{-1, 1, 1}, 0.0869922 + 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
 {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, -1, -1}, 0.0869922 - 0.770616 I},
 {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, -1, -1}, -0.00122183 - 0.770616 I}}
```

```
FiltPulse[8][0][4 T + 0.5 T]
```

```
0.770616
```

Now we build a receiver!! First generate the modulated sequence

```
Options[Modulator]
```

$$\left\{\texttt{StartingQuadrant} \to 0,\ \texttt{InitialState} \to \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\},\ \texttt{SamplingInterval} \to \frac{3}{26000000},\ \texttt{NumberOfCurves} \to 4,\ \texttt{ModulatingPulse} \to \texttt{LaurentC}\right\}$$

$$\frac{3}{26000000} \Big/ \texttt{T}$$

$$\frac{1}{32}$$

```
tom2 = Modulator[L][RandomBitSeq, NumberOfCurves → 2, ModulatingPulse → FiltPulse];

ListPlot[{Re[tom2], Im[tom2]} // Transpose, PlotJoined -> True]
```

```
- Graphics -
```

```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

```
Receiver[L][tom2, StartingQuadrant -> 4]
```

```
{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
 -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1,
 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1,
 -1, -1, 1, -1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1}
```

We now calculate the BER given the $\frac{E_b}{N_0}$ and L = 8. We write all the possible terms

$$\begin{aligned}
&J^{A_{0,N-5}}\ \big(\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}\ \texttt{Pulse[0][}\delta\texttt{T]}\ + \\
&\quad J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1})}\ \texttt{Pulse[0][T + }\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}\ \texttt{Pulse[0][2 T + }\delta\texttt{T]}\ + \\
&\quad J^{(\alpha_{N-4}+\alpha_{N-3})}\ \texttt{Pulse[0][3 T + }\delta\texttt{T]}\ +\ J^{\alpha_{N-4}}\ \texttt{Pulse[0][4 T + }\delta\texttt{T]}\ +\ \texttt{Pulse[0][5 T + }\delta\texttt{T]}\ + \\
&\quad J^{(-\alpha_{N-5})}\ \texttt{Pulse[0][6 T + }\delta\texttt{T]}\ +\ J^{(-\alpha_{N-5}-\alpha_{N-6})}\ \texttt{Pulse[0][7 T + }\delta\texttt{T]}\ + \\
&\quad J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}\ \texttt{Pulse[0][8 T + }\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N-\alpha_{N-1})}\ \texttt{Pulse[1][}\delta\texttt{T]}\ + \\
&\quad J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}-\alpha_{N-2})}\ \texttt{Pulse[1][T + }\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-3})}\ \texttt{Pulse[1][2 T + }\delta\texttt{T]}\ + \\
&\quad J^{(\alpha_{N-4}+\alpha_{N-3}-\alpha_{N-4})}\ \texttt{Pulse[1][3 T + }\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}-\alpha_{N-5})}\ \texttt{Pulse[1][4 T + }\delta\texttt{T]}\ + \\
&\quad J^{(-\alpha_{N-6})}\ \texttt{Pulse[1][5 T + }\delta\texttt{T]}\ +\ J^{(-\alpha_{N-5}+\alpha_{N-7})}\ \texttt{Pulse[1][6 T + }\delta\texttt{T]}\ \big)
\end{aligned}$$

We select the imaginary

$$\begin{aligned}
&J^{A_{0,N-5}}\ \big(\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}\ \texttt{Pulse[0][}\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}\ \texttt{Pulse[0][2 T + }\delta\texttt{T]}\ + \\
&\quad J^{\alpha_{N-4}}\ \texttt{Pulse[0][4 T + }\delta\texttt{T]}\ +\ J^{(-\alpha_{N-5})}\ \texttt{Pulse[0][6 T + }\delta\texttt{T]}\ + \\
&\quad J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}\ \texttt{Pulse[0][8 T + }\delta\texttt{T]}\ +\ J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}-\alpha_{N-2})}\ \texttt{Pulse[1][T + }\delta\texttt{T]}\ + \\
&\quad J^{(\alpha_{N-4}+\alpha_{N-3}-\alpha_{N-4})}\ \texttt{Pulse[1][3 T + }\delta\texttt{T]}\ +\ J^{(-\alpha_{N-6})}\ \texttt{Pulse[1][5 T + }\delta\texttt{T]}\ \big)
\end{aligned}$$

```
ModulationValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
 (1.` I)^(x0+x1+x2+x3+x4) Pulse[0][δT] + (1.` I)^(x2+x3+x4) Pulse[0][2 T + δT] +
  (1.` I)^x4 Pulse[0][4 T + δT] + (1.` I)^(-x5) Pulse[0][6 T + δT] +
  (1.` I)^(-x5-x6-x7) Pulse[0][8 T + δT] + (1.` I)^(x1+x3+x4) Pulse[1][T + δT] +
  (1.` I)^x3 Pulse[1][3 T + δT] + (1.` I)^(-x6) Pulse[1][5 T + δT]
```

```
ModulationValue[FiltPulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T] // N

-0.754091 I

ConvertToBitSeq[Depth_][Num_] :=
  Module[{bit, n},
  n = Num;
  ConvertNext := (bit = Mod[n, 2]; n = Floor[n / 2]; bit);
 ( Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 → -1}];

ConvertToBitSeq[8][2]

{-1, -1, -1, -1, -1, -1, 1, -1}

Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&;

Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&]

{{-1, -1, -1, -1, 1, -1, -1, -1},
 {-1, -1, -1, -1, 1, -1, -1, 1}, {-1, -1, -1, -1, 1, -1, 1, -1},
 {-1, -1, -1, -1, 1, -1, 1, 1}, {-1, -1, -1, -1, 1, 1, -1, -1},
 {-1, -1, -1, -1, 1, 1, -1, 1}, {-1, -1, -1, -1, 1, 1, 1, -1},
 {-1, -1, -1, -1, 1, 1, 1, 1}, {-1, -1, -1, 1, 1, -1, -1, -1},
 {-1, -1, -1, 1, 1, -1, -1, 1}, {-1, -1, -1, 1, 1, -1, 1, -1},
 {-1, -1, -1, 1, 1, -1, 1, 1}, {-1, -1, -1, 1, 1, 1, -1, -1}, {-1, -1, -1, 1, 1, 1, -1, 1},
 {-1, -1, -1, 1, 1, 1, 1, -1}, {-1, -1, -1, 1, 1, 1, 1, 1}, {-1, -1, 1, -1, 1, -1, -1, -1},
 {-1, -1, 1, -1, 1, -1, -1, 1}, {-1, -1, 1, -1, 1, -1, 1, -1},
 {-1, -1, 1, -1, 1, -1, 1, 1}, {-1, -1, 1, -1, 1, 1, -1, -1}, {-1, -1, 1, -1, 1, 1, -1, 1},
 {-1, -1, 1, -1, 1, 1, 1, -1}, {-1, -1, 1, -1, 1, 1, 1, 1}, {-1, -1, 1, 1, 1, -1, -1, -1},
 {-1, -1, 1, 1, 1, -1, -1, 1}, {-1, -1, 1, 1, 1, -1, 1, -1}, {-1, -1, 1, 1, 1, -1, 1, 1},
 {-1, -1, 1, 1, 1, 1, -1, -1}, {-1, -1, 1, 1, 1, 1, -1, 1}, {-1, -1, 1, 1, 1, 1, 1, -1},
 {-1, -1, 1, 1, 1, 1, 1, 1}, {-1, 1, -1, -1, 1, -1, -1, -1}, {-1, 1, -1, -1, 1, -1, -1, 1},
 {-1, 1, -1, -1, 1, -1, 1, -1}, {-1, 1, -1, -1, 1, -1, 1, 1},
 {-1, 1, -1, -1, 1, 1, -1, -1}, {-1, 1, -1, -1, 1, 1, -1, 1}, {-1, 1, -1, -1, 1, 1, 1, -1},
 {-1, 1, -1, -1, 1, 1, 1, 1}, {-1, 1, -1, 1, 1, -1, -1, -1}, {-1, 1, -1, 1, 1, -1, -1, 1},
 {-1, 1, -1, 1, 1, -1, 1, -1}, {-1, 1, -1, 1, 1, -1, 1, 1}, {-1, 1, -1, 1, 1, 1, -1, -1},
 {-1, 1, -1, 1, 1, 1, -1, 1}, {-1, 1, -1, 1, 1, 1, 1, -1}, {-1, 1, -1, 1, 1, 1, 1, 1},
 {-1, 1, 1, -1, 1, -1, -1, -1}, {-1, 1, 1, -1, 1, -1, -1, 1}, {-1, 1, 1, -1, 1, -1, 1, -1},
 {-1, 1, 1, -1, 1, -1, 1, 1}, {-1, 1, 1, -1, 1, 1, -1, -1}, {-1, 1, 1, -1, 1, 1, -1, 1},
 {-1, 1, 1, -1, 1, 1, 1, -1}, {-1, 1, 1, -1, 1, 1, 1, 1}, {-1, 1, 1, 1, 1, -1, -1, -1},
 {-1, 1, 1, 1, 1, -1, -1, 1}, {-1, 1, 1, 1, 1, -1, 1, -1}, {-1, 1, 1, 1, 1, -1, 1, 1},
 {-1, 1, 1, 1, 1, 1, -1, -1}, {-1, 1, 1, 1, 1, 1, -1, 1}, {-1, 1, 1, 1, 1, 1, 1, -1},
 {-1, 1, 1, 1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, -1, -1, -1}, {1, -1, -1, -1, 1, -1, -1, 1},
 {1, -1, -1, -1, 1, -1, 1, -1}, {1, -1, -1, -1, 1, -1, 1, 1},
 {1, -1, -1, -1, 1, 1, -1, -1}, {1, -1, -1, -1, 1, 1, -1, 1}, {1, -1, -1, -1, 1, 1, 1, -1},
 {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, 1, 1, -1, -1, -1}, {1, -1, -1, 1, 1, -1, -1, 1},
 {1, -1, -1, 1, 1, -1, 1, -1}, {1, -1, -1, 1, 1, -1, 1, 1}, {1, -1, -1, 1, 1, 1, -1, -1},
 {1, -1, -1, 1, 1, 1, -1, 1}, {1, -1, -1, 1, 1, 1, 1, -1}, {1, -1, -1, 1, 1, 1, 1, 1},
 {1, -1, 1, -1, 1, -1, -1, -1}, {1, -1, 1, -1, 1, -1, -1, 1}, {1, -1, 1, -1, 1, -1, 1, -1},
 {1, -1, 1, -1, 1, -1, 1, 1}, {1, -1, 1, -1, 1, 1, -1, -1}, {1, -1, 1, -1, 1, 1, -1, 1},
 {1, -1, 1, -1, 1, 1, 1, -1}, {1, -1, 1, -1, 1, 1, 1, 1}, {1, -1, 1, 1, 1, -1, -1, -1},
 {1, -1, 1, 1, 1, -1, -1, 1}, {1, -1, 1, 1, 1, -1, 1, -1}, {1, -1, 1, 1, 1, -1, 1, 1},
 {1, -1, 1, 1, 1, 1, -1, -1}, {1, -1, 1, 1, 1, 1, -1, 1}, {1, -1, 1, 1, 1, 1, 1, -1},
 {1, -1, 1, 1, 1, 1, 1, 1}, {1, 1, -1, -1, 1, -1, -1, -1}, {1, 1, -1, -1, 1, -1, -1, 1},
 {1, 1, -1, -1, 1, -1, 1, -1}, {1, 1, -1, -1, 1, -1, 1, 1}, {1, 1, -1, -1, 1, 1, -1, -1},
 {1, 1, -1, -1, 1, 1, -1, 1}, {1, 1, -1, -1, 1, 1, 1, -1}, {1, 1, -1, -1, 1, 1, 1, 1},
 {1, 1, -1, 1, 1, -1, -1, -1}, {1, 1, -1, 1, 1, -1, -1, 1}, {1, 1, -1, 1, 1, -1, 1, -1},
 {1, 1, -1, 1, 1, -1, 1, 1}, {1, 1, -1, 1, 1, 1, -1, -1}, {1, 1, -1, 1, 1, 1, -1, 1},
 {1, 1, -1, 1, 1, 1, 1, -1}, {1, 1, -1, 1, 1, 1, 1, 1}, {1, 1, 1, -1, 1, -1, -1, -1},
 {1, 1, 1, -1, 1, -1, -1, 1}, {1, 1, 1, -1, 1, -1, 1, -1}, {1, 1, 1, -1, 1, -1, 1, 1},
 {1, 1, 1, -1, 1, 1, -1, -1}, {1, 1, 1, -1, 1, 1, -1, 1}, {1, 1, 1, -1, 1, 1, 1, -1},
 {1, 1, 1, -1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, -1, -1, -1}, {1, 1, 1, 1, 1, -1, -1, 1},
 {1, 1, 1, 1, 1, -1, 1, -1}, {1, 1, 1, 1, 1, -1, 1, 1}, {1, 1, 1, 1, 1, 1, -1, -1},
 {1, 1, 1, 1, 1, 1, -1, 1}, {1, 1, 1, 1, 1, 1, 1, -1}, {1, 1, 1, 1, 1, 1, 1, 1}}
```

```
x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
   Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
  (-I)
```

```
{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
 0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}
```

Map$\left[\left(1/2 - \sigma/2\,\text{Erf}\left[\frac{\#}{\sigma}\right]\right)$**&, x1] // Apply[Plus, #]& // # / 128 &;**

Ber[σ_] := Evaluate[%48]

Ber[0.01]

0.495

Erf[Infinity]

1

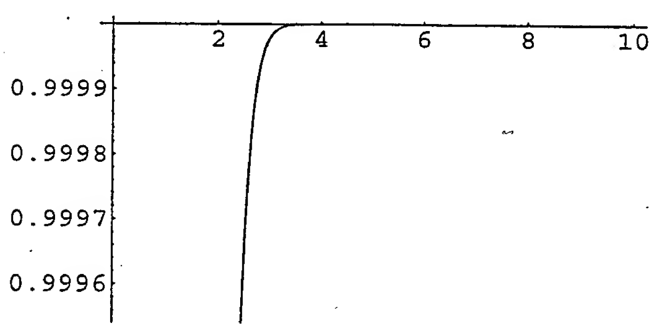
D[Erf[x], x]

$$\frac{2\,E^{-x^2}}{\sqrt{\pi}}$$

?Erf

Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error function erf(z1) - erf(z0).

Plot[Erf[x], {x, 0, 10}]



- Graphics -

?FiltPulse

Global`FiltPulse

```
T := 3 / 812500

Null

?? T

This is the symbol period

?? T

This is the symbol period

T := 3 / 812500

Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]
```

$J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}$ Pulse[0][δT] + $J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}$ Pulse[0][2 T + δT] + $J^{\alpha_{N-4}}$ Pulse[0][4 T + δT] + $J^{(-\alpha_{N-5})}$ Pulse[0][6 T + δT] + $J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}$ Pulse[0][8 T + δT] + $J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-1})}$ Pulse[1][T + δT] + $J^{(\alpha_{N-3})}$ Pulse[1][3 T + δT] + $J^{(-\alpha_{N-6})}$ Pulse[1][5 T + δT] /. { α_N → x0, α_{N-1} → x1, α_{N-2} → x2, α_{N-3} → x3, α_{N-4} → x4, α_{N-5} → x5, α_{N-6} → x6, α_{N-7} → x7}

$(1.\ I)^{x0+x1+x2+x3+x4}$ Pulse[0] [δT] + $(1.\ I)^{x2+x3+x4}$ Pulse[0] [2 T + δT] + $(1.\ I)^{x4}$ Pulse[0] [4 T + δT] + $(1.\ I)^{-x5}$ Pulse[0] [6 T + δT] + $(1.\ I)^{-x5-x6-x7}$ Pulse[0] [8 T + δT] + $(1.\ I)^{x1+x3+x4}$ Pulse[1] [T + δT] + $(1.\ I)^{x3}$ Pulse[1] [3 T + δT] + $(1.\ I)^{-x6}$ Pulse[1] [5 T + δT]

$J^{(\alpha_{N-3})}$ Pulse[1][3 T + δT] /. { α_N → x0, α_{N-1} → x1, α_{N-2} → x2, α_{N-3} → x3, α_{N-4} → x4, α_{N-5} → x5, α_{N-6} → x6, α_{N-7} → x7}

$(1.\ I)^{x3}$ Pulse[1] [3 T + δT]

$(1.`\ I)^{x3}$ Pulse[1] [3 T + δT]

$(1.\ I)^{x3}$ Pulse[1] [3 T + δT]

```
? J
```

$J = e^{j\phi}$

```
BeginPackage["LaurentFunctions`"]

T::usage = "This is the symbol period"

BT::usage = "This is the usual product"

h::usage = "This is the raw gaussian pulse"

Φ::usage = "This denotes modulation index Pi"

ψ::usage = "ψ[L,t] is Laurents function"

hFiltered::usage = "This is the filtered gaussian pulse"

PhaseAngle::usage = "This function takes some time to calculate. The following
   code will draw a graph of the functionPhaseFunction[t_] =N[ φL,t ];
phasepoints =Table[{t,PhaseFunction[t T]}//N,{t,0,L ,1/40}];
ListPlot[phasepoints,PlotJoined -> True]"

S ::usage = "S = Sin[Φ]"

J::usage = "J = e^(j Φ)"

C::usage = "C = Cos[Φ]"

M::usage = "M = 2^(L-1)"

ModulationIndex::usage = "ModulationIndex = h"

LaurentS::usage = "LaurentS[L][n][t] = Sin[ ψ[L,t + n T]]/ S"

LaurentC::usage = "LaurentC[L][K][t] is Laurents CK,t"

AlphaKI::usage = "AlphaKI[LL][K,i] is Laurents αK,i"

LaurentLK::usage = "LaurentLK[L][K] gives the support of CK,t"
```

The start of Modulator Definitions

```
ANKInitialStateSetUp::usage =
 "ANKInitialStateSetUp[L][K][InitBitSeq,AccumulatedPhase] sets up the sequence
   of prior states of AK,N that the modulator went thgough to get to the
   constelation point specified by  AccumulatedPhase which is really A0,0"

AKN::usage = "AKN[L][K][{State,AccumulatedPhase}] defines AK,N in terms of A0,N"

ModulatingPulse::usage =
 "The Pulse is assumed to have the following structure Pulse[L][K][t]"

NumberOfCurves::usage = "The number of pulses used by the modulator"

SamplingInterval::usage =
 "SamplingInterval is the interval between samples of the output of the modulator"

InitialState::usage = "InitialState is the set of bits that are assumed
   to be present before i.e {a-1,a-2, ...}"

StartingQuadrant::usage = "StartingQuadrant = A0,-1and is a number"

Modulator::usage = "Modulator[L][BitSeq,Opts] assumes the following
   default options StartingQuadrant → 0,InitialState → Table[1,{i,
   1,20}],SamplingInterval → T/32,NumberOfCurves → 4,ModulatingPulse→
   LaurentC . The Pulse is assumed to have the following structure Pulse[L][K][t]"
```

Start of the Receiver Functions

```
FiltPulse::usage = "The default pulse and is called by FiltPulse[L][K][t]"

SyncSample::usage = "Given that the sampling interval is T/32,then sync
   sample has rang -16 to 16 and this moves the point used to demodulate "

Receiver::usage = " Receiver[L][InputSeq,Opts] assumes the following
   default options {StartingQuadrant→0,InitialState→{1,1,1,1,1,1,1,1,
   1,1,1,1,1,1,1,1,1,1,1,1},SamplingInterval -> T/32,ModulatingPulse →
   FiltPulse,SyncSample -> 0,NumberOfCurves -> 2   "

ReceiverProper::usage = "ReceiverProper[L][ StartingQuadrant, InitialState,
   SamplingInterval,ModulatingPulse,SyncSample,NumberOfCurves,InputSeq]
   is called by Receiver after all the options have been resolved"

Begin["`Private`"]
```

$$\sigma := \frac{\sqrt{\mathrm{Log}[2]}}{2\,\pi\,\mathrm{BT}}$$

$$\mathrm{h}[\mathrm{t\_}] := \frac{\mathrm{Exp}\left[-\frac{t^2}{2\,\sigma^2\,T^2}\right]}{\sqrt{2\,\pi}\,\sigma\,T}$$

Ideally we would and did define the effect of the convolution of h[t] by the formula below. However, this version of *Mathematica* gives an error.

$$\mathrm{h}_{fil}[\mathrm{t\_}] := \mathrm{Release}\left[\mathrm{Module}\left[\{\tau\},\ \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\mathrm{h}[t-\tau]}{T}\,d\tau\right]\right]$$

$$\mathrm{hFiltered}[\mathrm{PulseWidth\_}][\mathrm{t\_}] := \mathrm{Module}\Big[\{x1,\ x2,\ x3\},\ x1 = (\mathrm{N}[\#1,\ 40]\&)\Big[\mathrm{Table}\Big[\Big\{t1,\ \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\mathrm{h}[t1-\tau]}{T}\,d\tau\Big\},\ \Big\{t1,\ -\frac{\mathrm{PulseWidth}}{2},\ \frac{\mathrm{PulseWidth}}{2},\ \frac{T}{20}\Big\}\Big]\Big];\ x2 = \mathrm{Interpolation}[x1];\ x2[t]\Big]$$

```
Φ := N[ModulationIndex π]

C := Cos[Φ];
S := Sin[Φ];
J := ( e^(i Φ) // Chop);
M := 2^(L-1);

PhaseAngle[L_][t_] /; t ≤ 0 := 0

PhaseAngle[L_][t_] /; t ≥ L T := Φ

PhaseAngle[L_][t_] :=
 PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, x1 = hFiltered[3 L T][t1 - L T/2];
   x2 = Table[{t2, Φ ∫_{-L T}^{t2} Evaluate[x1] dt1}, {t2, 0, L T, T/100}]; Interpolation[x2][t]]

ψ[L_, t_] /; 0 < t < L T := PhaseAngle[L][t]

ψ[L_, t_] /; 2 L T > t ≥ L T := Φ - PhaseAngle[L][t - L T]
```

We need to put this to avoid the function being extrapolated

```
ψ[L_ , t_] /; ! (0 < t < L T) && ! (2 L T > t ≥ L T) := 0

LaurentS[L_][n_][t_] := Sin[ ψ[L, t + n T]] / S
```

```
AlphaKI[LL_][K_, i_] /; (0 < i < LL) && (0 <= K < 2^(LL-1)) :=
 Module[{x1, x2, x3, KNum}, x1 := (x2 = Mod[KNum, 2]; KNum = (KNum - x2)/2; x2);
  KNum = K; x3 = Table[x1, {ii, 0, LL - 1}]; x3[[i]]]

LaurentLK[L_][K_] :=
 Module[{x1}, x1 = Table[L (2 - AlphaKI[L][K, ii]) - ii, {ii, 1, L - 1}]; Min[x1]]

LaurentC[L_][K_][t_] /; 0 <= K < 2^L :=
 LaurentS[L][0][t] Product[LaurentS[L][ii + L AlphaKI[L][K, ii]][t], {ii, 1, L-1}]
```

The start of the modulator function

```
ANKInitialStateSetUp[L_][K_][InitBitSeq_, AccumulatedPhase_] :=
         Module[{x1, x2, x3, x4, x5, acuphase, initbitseq},
  initbitseq = InitBitSeq;
  acuphase = AccumulatedPhase;
   UpdateSeq :=
   Module[{}, x1 = acuphase - Sum[initbitseq[[i ]] AlphaKI[L][K, i], {i, 1, L - 1}];
    acuphase = acuphase - First[initbitseq]; initbitseq = Rest[ initbitseq]; x1];
  Table[ UpdateSeq, {i, 1, LaurentLK[L][K]}]]

AKN[L_][K_][{State_, AccumulatedPhase_}] :=
 AccumulatedPhase - Sum[State[[i + 1]] AlphaKI[L][K, i], {i, 1, L - 1}]

Options[Modulator] := {StartingQuadrant → 0, InitialState → Table[1, {i, 1, 20}],
  SamplingInterval → T / 32, NumberOfCurves → 4, ModulatingPulse → LaurentC}

Modulator[L_][BitSeq_, Opts___] :=
 Module[
  {x1, x2, x3, x4, x5, x6, state, AccumalatedPhase, seq, AKNState, Curves, Pulse},
x1 = SamplingInterval /. {Opts} /. Options[Modulator];
state = InitialState /. {Opts} /. Options[Modulator];
x3 = StartingQuadrant /. {Opts} /. Options[Modulator];
x4 = SamplingInterval /. {Opts} /. Options[Modulator];
Pulse = ModulatingPulse /. {Opts} /. Options[Modulator];
Curves = (NumberOfCurves /. {Opts} /. Options[Modulator] ) - 1;
AccumulatedPhase = x3 ;
seq = BitSeq;
Table[
   AKNState[K] = ANKInitialStateSetUp[L][K][state, AccumulatedPhase], {K, 0, Curves}];
x5 := Module[{}, state = Join[{First[seq]}, Drop[state, -1]];
AccumulatedPhase = AccumulatedPhase + First[seq];
seq = Rest[seq];
Table[AKNState[K] = Join[{AKN[L][K][{state, AccumulatedPhase}]},
       Drop[AKNState[K], -1]], {K, 0, Curves}];
x6[τ_] = Sum[Sum[ (J)^AKNState[K][[i + 1]] Pulse[L][K][τ + i T],
       {i, 0, LaurentLK[L][K] - 1}], {K, 0, Curves}];
Table[x6[τ], {τ, 0, T - x4, x4}]];
 Table[x5, {kk , 1, Length[BitSeq]}] // Flatten]

Options[Receiver] := {StartingQuadrant → 0,
  InitialState → {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, SamplingInter
  ModulatingPulse → FiltPulse, SyncSample -> 0, NumberOfCurves -> 2};

Receiver[L_][InputSeq_, Opts___] :=
  Module[{x1, x2, x3, x4, x5, x6},
  x1 = StartingQuadrant /. {Opts} /. Options[Receiver];
  x2 = InitialState /. {Opts} /. Options[Receiver];
  x3 = SamplingInterval /. {Opts} /. Options[Receiver];
  x4 = ModulatingPulse /. {Opts} /. Options[Receiver];
  x5 = SyncSample /. {Opts} /. Options[Receiver];
  x6 = NumberOfCurves /. {Opts} /. Options[Receiver];
ReceiverProper[L][x1, x2, x3, x4, x5, x6, InputSeq]]
```

```
ReceiverProper[L_][ StartingQuadrant_, InitialState_, SamplingInterval_,
  ModulatingPulse_, SyncSample_, NumberOfCurves_, InputSeq_] :=
     Module[{x1, sgn, ReceivedSeq,  ExpectedValue, seq, ReceiveNext, D, 𝒜},
  x1 = T / SamplingInterval;
  ReceivedSeq = Partition[InputSeq, x1] // Transpose // #[[SyncSample + 1]]&;
  ExpectedValue =
   ModulatingPulse[L][0][(LaurentLK[L][0] / 2) T + SyncSample SamplingInterval];
  𝒜 = StartingQuadrant;
  sgn = 0;
  D = {};
  seq = ReceivedSeq;
  ReceiveNext :=
  Module[{x1, x2},
    x1 = ( (-1)^sgn J^𝒜 First[seq]) // Im;
   If[Abs[x1  - ExpectedValue ] <= Abs[x1  + ExpectedValue ],
     D  = Join[ D, {1}];  𝒜 =  𝒜 + 1,  D  = Join[ D, {-1}];  𝒜 =  𝒜 - 1 ];
    seq = Rest[seq]; sgn = Mod[sgn + 1, 2]];
 Table[ReceiveNext, {i, 1, Length[ReceivedSeq]}];
  D]

End[]

EndPackage[]
```

Needs["LaurentFunctions`"]

```
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
   PhaseAngle[L_][t_] :→ (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).
```

Needs["LaurentNotationTest`"]

```
Needs::nocont : Context LaurentNotationTest` was not created when Needs was evaluated.
```

Information on the functions used can be obtained using help.

Names["LaurentFunctions`*"]

```
{AKN, AlphaKI, ANKInitialStateSetUp, BT, FiltPulse, h, hFiltered, InitialState, J,
 LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
 NumberOfCurves, PhaseAngle, Receiver, ReceiverProper, S, SamplingInterval,
 StartingQuadrant, SyncSample, T, C, Φ, ψ}
```

T := $\frac{3}{812500}$
BT := 0.3

ModulationIndex := $\frac{1}{2}$

<< ModulatorData.m;

RandomBitSeq

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

We show that it is possible to build a ~~receiver~~ mobile telephone to receive ~~pulses that have~~ a signal modulated using ~~pulses~~ supperposition method of Laurent - but with different pulses that have other desirable properties ~~like~~ like low bandwidth etc.

```
T
```

$$\frac{3}{812500}$$

$$S_{NT+\Delta T} = \sum_{K=0}^{M-1} \sum_{n'=0}^{L_K-1} J^{A_{K,N-n'}} C_{K,n'T+\Delta T}$$

With L = 8, and M = 2 we can utilise only the main values. It so happens that when K = 1, the dominant value occurs at 2.5T in the filtered pulse. The three dominant values when K =0 occurs at 3.5T, 4.5T and 5.5T. The value of the pulse at 6.5T is smaller than the value of the pulse with K = 1 at 2.5 T. Thus the expression becomes

$$S_{NT+\Delta T} = \sum_{K=0}^{1} \sum_{n'=3}^{5} J^{A_{K,N-n'}} C_{K,n'T+\Delta T}$$

$$\tilde{S}_{NT+\Delta T} = J^{\lambda_{0,N-3}} \text{Pulse}[0][3T+\delta T] + J^{\lambda_{0,N-4}} \text{Pulse}[0][4T+\delta T] + J^{\lambda_{0,N-5}} \text{Pulse}[0][5T+\delta T] + J^{\lambda_{1,N-2}} \text{Pulse}[1][2T+\delta T]$$

$$\tilde{S}_{NT+\Delta T} = J^{\lambda_{0,N-3}} \text{Pulse}[0][3T+\delta T] + J^{\lambda_{0,N-4}} \text{Pulse}[0][4T+\delta T] + J^{\lambda_{0,N-5}} \text{Pulse}[0][5T+\delta T] + J^{(\lambda_{0,N-2}+\alpha_{N-3})} \text{Pulse}[1][2T+\delta T]$$

Given that α_{N-3} has been already decided, we can precalculate the possibilities and store them.

$$\tilde{S}_{NT+\Delta T} = J^{\lambda_{0,N-5}} \left(J^{(\alpha_{N-4}+\alpha_{N-3})} \text{Pulse}[0][3T+\delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T+\delta T] + \text{Pulse}[0][5T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-3})} \text{Pulse}[1][2T+\delta T] \right)$$

But $J^2 = 1$ and so we get

$$\tilde{S}_{NT+\Delta T} = J^{\lambda_{0,N-5}} \left(J^{(\alpha_{N-4}+\alpha_{N-3})} \text{Pulse}[0][3T+\delta T] + J^{\alpha_{N-4}} \text{Pulse}[0][4T+\delta T] + \text{Pulse}[0][5T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-2})} \text{Pulse}[1][2T+\delta T] \right)$$

```
LookUpTable[Pulse_, δT_] :=
    Module[{x1, x2, x3, x4},
  x1 = {{-1, -1, -1}, {-1, -1, 1},
     {-1, 1, -1}, {-1, 1, 1}, {1, -1, -1}, {1, -1, 1}, {1, 1, -1}, {1, 1, 1}};
  x2[bitseq_] :=
   ( J^(bitseq[[3]] + bitseq[[2]]) Pulse[0][3 T + δT]  +  J^bitseq[[3]] Pulse[0][4 T + δT] +
     Pulse[0][5 T + δT]  +  J^(bitseq[[3]] + bitseq[[1]]) Pulse[1][2 T + δT]);
 {x1,  Map[x2[#]&, x1]} // Transpose]

tab = LookUpTable[FiltPulse[8], 0.5 T]

{{{-1, -1, -1}, -0.00122183 - 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
 {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, 1, 1}, 0.0869922 + 0.770616 I},
 {{1, -1, -1}, 0.0869922 - 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
 {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, 1, 1}, -0.00122183 + 0.770616 I}}

Sort[tab, #1[[1, 3]] > #2[[1, 3]]&]

{{{1, 1, 1}, -0.00122183 + 0.770616 I}, {{1, -1, 1}, 0.614125 + 0.770616 I},
 {{-1, 1, 1}, 0.0869922 + 0.770616 I}, {{-1, -1, 1}, 0.702339 + 0.770616 I},
 {{1, 1, -1}, 0.702339 - 0.770616 I}, {{1, -1, -1}, 0.0869922 - 0.770616 I},
 {{-1, 1, -1}, 0.614125 - 0.770616 I}, {{-1, -1, -1}, -0.00122183 - 0.770616 I}}

FiltPulse[8][0][4 T + 0.5 T]

0.770616
```

Now we build a receiver!! First generate the modulated sequence

```
Options[Modulator]
```

$$\{\text{StartingQuadrant} \to 0,\ \text{InitialState} \to \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\},\ \text{SamplingInterval} \to \frac{3}{26000000},\ \text{NumberOfCurves} \to 4,\ \text{ModulatingPulse} \to \text{LaurentC}\}$$

$$\frac{3}{26000000} \Big/ T$$

$$\frac{1}{32}$$

```
tom2 = Modulator[L][RandomBitSeq, NumberOfCurves → 2, ModulatingPulse → FiltPulse];
```

```
ListPlot[{Re[tom2], Im[tom2]} // Transpose, PlotJoined -> True]
```

- Graphics -

```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

```
Receiver[L][tom2, StartingQuadrant -> 4]
```

```
{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
 -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1,
 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1,
 -1, -1, 1, -1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1}
```

We now calculate the BER given the $\frac{E_b}{N_0}$ and L = 8. We write all the possible terms

$$\begin{aligned} J^{\lambda_{0,N-5}} \Big(& J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}\,\text{Pulse}[0][\delta T] + \\ & J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1})}\,\text{Pulse}[0][T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}\,\text{Pulse}[0][2T+\delta T] + \\ & J^{(\alpha_{N-4}+\alpha_{N-3})}\,\text{Pulse}[0][3T+\delta T] + J^{\alpha_{N-4}}\,\text{Pulse}[0][4T+\delta T] + \text{Pulse}[0][5T+\delta T] + \\ & J^{(-\alpha_{N-5})}\,\text{Pulse}[0][6T+\delta T] + J^{(-\alpha_{N-5}-\alpha_{N-6})}\,\text{Pulse}[0][7T+\delta T] + \\ & J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}\,\text{Pulse}[0][8T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N-\alpha_{N-1})}\,\text{Pulse}[1][\delta T] + \\ & J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}-\alpha_{N-2})}\,\text{Pulse}[1][T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-3})}\,\text{Pulse}[1][2T+\delta T] + \\ & J^{(\alpha_{N-4}+\alpha_{N-3}-\alpha_{N-4})}\,\text{Pulse}[1][3T+\delta T] + J^{(\alpha_{N-4}-\alpha_{N-5})}\,\text{Pulse}[1][4T+\delta T] + \\ & J^{(-\alpha_{N-6})}\,\text{Pulse}[1][5T+\delta T] + J^{(-\alpha_{N-5}+\alpha_{N-7})}\,\text{Pulse}[1][6T+\delta T] \Big) \end{aligned}$$

We select the imaginary

$$\begin{aligned} J^{\lambda_{0,N-5}} \Big(& J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}\,\text{Pulse}[0][\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}\,\text{Pulse}[0][2T+\delta T] + \\ & J^{\alpha_{N-4}}\,\text{Pulse}[0][4T+\delta T] + J^{(-\alpha_{N-5})}\,\text{Pulse}[0][6T+\delta T] + \\ & J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}\,\text{Pulse}[0][8T+\delta T] + J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}-\alpha_{N-2})}\,\text{Pulse}[1][T+\delta T] + \\ & J^{(\alpha_{N-4}+\alpha_{N-3}-\alpha_{N-4})}\,\text{Pulse}[1][3T+\delta T] + J^{(-\alpha_{N-6})}\,\text{Pulse}[1][5T+\delta T] \Big) \end{aligned}$$

```
ModulationValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
```

$$\begin{aligned} & (1.`\,I)^{x0+x1+x2+x3+x4}\,\text{Pulse}[0][\delta T] + (1.`\,I)^{x2+x3+x4}\,\text{Pulse}[0][2T+\delta T] + \\ & (1.`\,I)^{x4}\,\text{Pulse}[0][4T+\delta T] + (1.`\,I)^{-x5}\,\text{Pulse}[0][6T+\delta T] + \\ & (1.`\,I)^{-x5-x6-x7}\,\text{Pulse}[0][8T+\delta T] + (1.`\,I)^{x1+x3+x4}\,\text{Pulse}[1][T+\delta T] + \\ & (1.`\,I)^{x3}\,\text{Pulse}[1][3T+\delta T] + (1.`\,I)^{-x6}\,\text{Pulse}[1][5T+\delta T] \end{aligned}$$

```
ModulationValue[FiltPulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T] // N

-0.754091 I

ConvertToBitSeq[Depth_][Num_] :=
  Module[{bit, n},
  n = Num;
  ConvertNext := (bit = Mod[n, 2]; n = Floor[n / 2]; bit);
 ( Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 → -1}];

ConvertToBitSeq[8][2]

{-1, -1, -1, -1, -1, -1, 1, -1}

Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&;

Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&]

{{-1, -1, -1, -1, 1, -1, -1, -1},
 {-1, -1, -1, -1, 1, -1, -1, 1}, {-1, -1, -1, -1, 1, -1, 1, -1},
 {-1, -1, -1, -1, 1, -1, 1, 1}, {-1, -1, -1, -1, 1, 1, -1, -1},
 {-1, -1, -1, -1, 1, 1, -1, 1}, {-1, -1, -1, -1, 1, 1, 1, -1},
 {-1, -1, -1, -1, 1, 1, 1, 1}, {-1, -1, -1, 1, 1, -1, -1, -1},
 {-1, -1, -1, 1, 1, -1, -1, 1}, {-1, -1, -1, 1, 1, -1, 1, -1},
 {-1, -1, -1, 1, 1, -1, 1, 1}, {-1, -1, -1, 1, 1, 1, -1, -1}, {-1, -1, -1, 1, 1, 1, -1, 1},
 {-1, -1, -1, 1, 1, 1, 1, -1}, {-1, -1, -1, 1, 1, 1, 1, 1}, {-1, -1, 1, -1, 1, -1, -1, -1},
 {-1, -1, 1, -1, 1, -1, -1, 1}, {-1, -1, 1, -1, 1, -1, 1, -1},
 {-1, -1, 1, -1, 1, -1, 1, 1}, {-1, -1, 1, -1, 1, 1, -1, -1}, {-1, -1, 1, -1, 1, 1, -1, 1},
 {-1, -1, 1, -1, 1, 1, 1, -1}, {-1, -1, 1, -1, 1, 1, 1, 1}, {-1, -1, 1, 1, 1, -1, -1, -1},
 {-1, -1, 1, 1, 1, -1, -1, 1}, {-1, -1, 1, 1, 1, -1, 1, -1}, {-1, -1, 1, 1, 1, -1, 1, 1},
 {-1, -1, 1, 1, 1, 1, -1, -1}, {-1, -1, 1, 1, 1, 1, -1, 1}, {-1, -1, 1, 1, 1, 1, 1, -1},
 {-1, -1, 1, 1, 1, 1, 1, 1}, {-1, 1, -1, -1, 1, -1, -1, -1}, {-1, 1, -1, -1, 1, -1, -1, 1},
 {-1, 1, -1, -1, 1, -1, 1, -1}, {-1, 1, -1, -1, 1, -1, 1, 1},
 {-1, 1, -1, -1, 1, 1, -1, -1}, {-1, 1, -1, -1, 1, 1, -1, 1}, {-1, 1, -1, -1, 1, 1, 1, -1},
 {-1, 1, -1, -1, 1, 1, 1, 1}, {-1, 1, -1, 1, 1, -1, -1, -1}, {-1, 1, -1, 1, 1, -1, -1, 1},
 {-1, 1, -1, 1, 1, -1, 1, -1}, {-1, 1, -1, 1, 1, -1, 1, 1}, {-1, 1, -1, 1, 1, 1, -1, -1},
 {-1, 1, -1, 1, 1, 1, -1, 1}, {-1, 1, -1, 1, 1, 1, 1, -1}, {-1, 1, -1, 1, 1, 1, 1, 1},
 {-1, 1, 1, -1, 1, -1, -1, -1}, {-1, 1, 1, -1, 1, -1, -1, 1}, {-1, 1, 1, -1, 1, -1, 1, -1},
 {-1, 1, 1, -1, 1, -1, 1, 1}, {-1, 1, 1, -1, 1, 1, -1, -1}, {-1, 1, 1, -1, 1, 1, -1, 1},
 {-1, 1, 1, -1, 1, 1, 1, -1}, {-1, 1, 1, -1, 1, 1, 1, 1}, {-1, 1, 1, 1, 1, -1, -1, -1},
 {-1, 1, 1, 1, 1, -1, -1, 1}, {-1, 1, 1, 1, 1, -1, 1, -1}, {-1, 1, 1, 1, 1, -1, 1, 1},
 {-1, 1, 1, 1, 1, 1, -1, -1}, {-1, 1, 1, 1, 1, 1, -1, 1}, {-1, 1, 1, 1, 1, 1, 1, -1},
 {-1, 1, 1, 1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, -1, -1, -1}, {1, -1, -1, -1, 1, -1, -1, 1},
 {1, -1, -1, -1, 1, -1, 1, -1}, {1, -1, -1, -1, 1, -1, 1, 1},
 {1, -1, -1, -1, 1, 1, -1, -1}, {1, -1, -1, -1, 1, 1, -1, 1}, {1, -1, -1, -1, 1, 1, 1, -1},
 {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, 1, 1, -1, -1, -1}, {1, -1, -1, 1, 1, -1, -1, 1},
 {1, -1, -1, 1, 1, -1, 1, -1}, {1, -1, -1, 1, 1, -1, 1, 1}, {1, -1, -1, 1, 1, 1, -1, -1},
 {1, -1, -1, 1, 1, 1, -1, 1}, {1, -1, -1, 1, 1, 1, 1, -1}, {1, -1, -1, 1, 1, 1, 1, 1},
 {1, -1, 1, -1, 1, -1, -1, -1}, {1, -1, 1, -1, 1, -1, -1, 1}, {1, -1, 1, -1, 1, -1, 1, -1},
 {1, -1, 1, -1, 1, -1, 1, 1}, {1, -1, 1, -1, 1, 1, -1, -1}, {1, -1, 1, -1, 1, 1, -1, 1},
 {1, -1, 1, -1, 1, 1, 1, -1}, {1, -1, 1, -1, 1, 1, 1, 1}, {1, -1, 1, 1, 1, -1, -1, -1},
 {1, -1, 1, 1, 1, -1, -1, 1}, {1, -1, 1, 1, 1, -1, 1, -1}, {1, -1, 1, 1, 1, -1, 1, 1},
 {1, -1, 1, 1, 1, 1, -1, -1}, {1, -1, 1, 1, 1, 1, -1, 1}, {1, -1, 1, 1, 1, 1, 1, -1},
 {1, -1, 1, 1, 1, 1, 1, 1}, {1, 1, -1, -1, 1, -1, -1, -1}, {1, 1, -1, -1, 1, -1, -1, 1},
 {1, 1, -1, -1, 1, -1, 1, -1}, {1, 1, -1, -1, 1, -1, 1, 1}, {1, 1, -1, -1, 1, 1, -1, -1},
 {1, 1, -1, -1, 1, 1, -1, 1}, {1, 1, -1, -1, 1, 1, 1, -1}, {1, 1, -1, -1, 1, 1, 1, 1},
 {1, 1, -1, 1, 1, -1, -1, -1}, {1, 1, -1, 1, 1, -1, -1, 1}, {1, 1, -1, 1, 1, -1, 1, -1},
 {1, 1, -1, 1, 1, -1, 1, 1}, {1, 1, -1, 1, 1, 1, -1, -1}, {1, 1, -1, 1, 1, 1, -1, 1},
 {1, 1, -1, 1, 1, 1, 1, -1}, {1, 1, -1, 1, 1, 1, 1, 1}, {1, 1, 1, -1, 1, -1, -1, -1},
 {1, 1, 1, -1, 1, -1, -1, 1}, {1, 1, 1, -1, 1, -1, 1, -1}, {1, 1, 1, -1, 1, -1, 1, 1},
 {1, 1, 1, -1, 1, 1, -1, -1}, {1, 1, 1, -1, 1, 1, -1, 1}, {1, 1, 1, -1, 1, 1, 1, -1},
 {1, 1, 1, -1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, -1, -1, -1}, {1, 1, 1, 1, 1, -1, -1, 1},
 {1, 1, 1, 1, 1, -1, 1, -1}, {1, 1, 1, 1, 1, -1, 1, 1}, {1, 1, 1, 1, 1, 1, -1, -1},
 {1, 1, 1, 1, 1, 1, -1, 1}, {1, 1, 1, 1, 1, 1, 1, -1}, {1, 1, 1, 1, 1, 1, 1, 1}}
```

```
x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
   Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
  (-I)
```

```
{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
 0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}
```

Map[$\left(1/2 - \sigma/2\,\mathrm{Erf}\left[\frac{\#}{\sigma}\right]\right)$ &, x1] // Apply[Plus, #]& // # / 128 &;

```
Ber[σ_] := Evaluate[%48]
```

```
Ber[0.01]
```

```
0.495
```

```
Erf[Infinity]
```

```
1
```

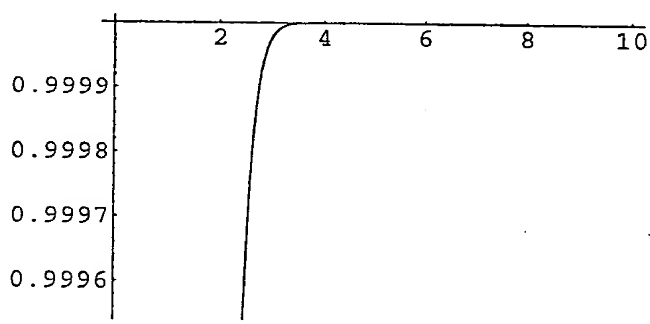
```
D[Erf[x], x]
```

$$\frac{2\,E^{-x^2}}{\sqrt{\pi}}$$

```
?Erf
```

```
Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error
  function erf(z1) - erf(z0).
```

```
Plot[Erf[x], {x, 0, 10}]
```




```
- Graphics -
```

```
?FiltPulse
```

```
Global`FiltPulse
```

```
T := 3 / 812500

Null

?? T

This is the symbol period

?? T

This is the symbol period

T := 3 / 812500

Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]
```

$J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2}+\alpha_{N-1}+\alpha_N)}$ Pulse[0][δT] + $J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-2})}$ Pulse[0][2 T + δT] + $J^{\alpha_{N-4}}$ Pulse[0][4 T + δT] + $J^{(-\alpha_{N-5})}$ Pulse[0][6 T + δT] + $J^{(-\alpha_{N-5}-\alpha_{N-6}-\alpha_{N-7})}$ Pulse[0][8 T + δT] + $J^{(\alpha_{N-4}+\alpha_{N-3}+\alpha_{N-1})}$ Pulse[1][T + δT] + $J^{(\alpha_{N-3})}$ Pulse[1][3 T + δT] + $J^{(-\alpha_{N-6})}$ Pulse[1][5 T + δT] /. { $\alpha_N \to$ x0, $\alpha_{N-1} \to$ x1, $\alpha_{N-2} \to$ x2, $\alpha_{N-3} \to$ x3, $\alpha_{N-4} \to$ x4, $\alpha_{N-5} \to$ x5, $\alpha_{N-6} \to$ x6, $\alpha_{N-7} \to$ x7}

(1. I)$^{x0+x1+x2+x3+x4}$ Pulse[0][δT] + (1. I)$^{x2+x3+x4}$ Pulse[0][2 T + δT] + (1. I)^{x4} Pulse[0][4 T + δT] + (1. I)$^{-x5}$ Pulse[0][6 T + δT] + (1. I)$^{-x5-x6-x7}$ Pulse[0][8 T + δT] + (1. I)$^{x1+x3+x4}$ Pulse[1][T + δT] + (1. I)^{x3} Pulse[1][3 T + δT] + (1. I)$^{-x6}$ Pulse[1][5 T + δT]

$J^{(\alpha_{N-3})}$ Pulse[1][3 T + δT] /. { $\alpha_N \to$ x0, $\alpha_{N-1} \to$ x1, $\alpha_{N-2} \to$ x2, $\alpha_{N-3} \to$ x3, $\alpha_{N-4} \to$ x4, $\alpha_{N-5} \to$ x5, $\alpha_{N-6} \to$ x6, $\alpha_{N-7} \to$ x7}

(1. I)^{x3} Pulse[1][3 T + δT]

(1.` I)^{x3} Pulse[1][3 T + δT]

(1. I)^{x3} Pulse[1][3 T + δT]

? J

J = $e^{i\phi}$